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Phonon-assisted two-photon absorption in the presence of a dc-field: the nonlinear Franz–Keldysh effect in indirect gap semiconductors

Hernando Garcia[1] **and Ramki Kalyanaraman**[2]

[1] Department of Physics, Southern Illinois University, Edwardsville, IL 62026, USA
[2] Department of Physics, Washington University in St Louis, St Louis, MO 63130, USA

E-mail: hgarcia@siue.edu and ramkik@wustl.edu

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Abstract

The two-photon absorption coefficient of an indirect gap semiconductor (phonon-assisted two-photon absorption) in the presence of a strong dc-electric field applied perpendicular to the direction of propagation of the optical field is calculated using the formalism developed elsewhere (Aspnes 1996 *Phys. Rev. B*. **147** 554). We show that depending on the type of transition (i.e., allowed–allowed, allowed–forbidden or forbidden–forbidden), the absorption coefficient followed different dispersion relations. In the limit of a weak electric field, we recovered results previously calculated using perturbation theory. In the strong dc-field regime, we found that below the rescaled energy gap given by E_g/N, where N is the number of photons, the tunnelling effect is present, but to our surprise, above the rescaled gap, the Franz–Keldysh oscillations are present only for the allowed–allowed transition. This absence of the oscillations in the allowed–forbidden and forbidden–forbidden transitions is possible due to the weak coupling of the tails of the electron and hole wavefunctions.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The effect of a dc-electric field on the absorption coefficient of solids and particularly in semiconductors has been extensively studied [1–5]. The main characteristics of the effect include the appearance of an exponential tail below the band edge reminiscent of the tunnelling effect and the development of an oscillatory behaviour (Franz–Keldysh oscillations) above the fundamental absorption edge E_g [6, 7]. This complex phenomenon has been used to characterize the band structure and to identify critical points in semiconductors [8, 9].

In the case of two-photon absorption, we have developed a formalism that expresses in closed form the two-photon absorption coefficient in the presence of a dc-field [10, 11] for a

direct band-gap semiconductor. One of the main features of the formalism is the universality of the Franz–Keldysh effect with respect to a rescaled energy gap given by E_g/N, where N is the number of photons involved in the process and E_g is the energy gap.

For the case of indirect semiconductors such as Si and Ge, there has been a lack of systematic studies on the effect of an electric field on the two-photon absorption coefficient, although there have been studies in the absence of an electric field [12–14]. The study of the two-photon absorption coefficient in an indirect gap semiconductor like silicon is important because it is widely used in a variety of optical devices, including, in silicon-based lasers [16]. Moreover, it is well known that two-photon absorption can place fundamental limits on all waveguide-based optical switching devices [17]. Furthermore, it is also known that the dominant absorption mechanism at telecommunication wavelengths ($\lambda = 1.5\ \mu$m) is two-photon absorption [18]. Therefore, the knowledge of the spectral dependence and electric field influence on the two-photon absorption is of significant interest for device optimization and design.

We present here the calculation of the degenerate nonlinear absorption coefficient for the case of $N = 2$ under a very strong dc-electric field for an indirect gap semiconductor. Our result shows agreement with previous calculations for allowed–forbidden and forbidden–forbidden transitions with no electric field present [12, 13, 19]. When the electric field is turned on, the two-photon absorption shows a tail development below the rescaled indirect gap, but to our surprise above the $E_g/2$ the typical Franz–Keldysh oscillations are completely absent for allowed–forbidden and forbidden–forbidden transitions.

In section 2 of this paper, we will present the formal theory of electro-absorption for an indirect gap semiconductor. In section 3, we present calculations for allowed–allowed (a–a), allowed–forbidden (a–f) and forbidden–forbidden (f–f) transitions in the absence of an electric field. In section 4, we introduce the electric field and calculate the electro-absorption coefficient for a–a, a–f and f–f transitions. In section 5, we propose a new spectroscopy technique based on white-light continuum Z-scan and electric field modulation to study bulk properties of semiconductors in the nonlinear regime.

2. Theory

The effect of the electric field in indirect transitions in semiconductors is well established with the early calculations reported by Penchina [20], Chester [21] and Yacoby [3]. The momentum conservation in an indirect transition requires the emission or absorption of a lattice vibration (phonon). Therefore, an indirect transition can be seen as a phonon-assisted electronic transition from the maximum of the valence band to a minimum in the conduction band occurring between different points in **k**-space. Aspnes [9, 15] showed how to calculate the absorption coefficient in the effective mass approximation (EMA) when scattering by a phonon is involved. In the case of phonon-assisted transition, the centre-of-mass (CM) energy of the electron–hole system need no longer be restricted to zero since the phonon is able to absorb momentum. Therefore, the absorption coefficient can be estimated as follows: start with the direct transition theory and make an additional sum over the centre-of-mass wave vector; this sum can be expressed as an integral over the CM as

$$\sum_{\mathrm{cm}} \rightarrow \frac{\left(2m_\mu^3\right)^{1/2}}{2\pi^2\hbar^3} \int E_{\mathrm{cm}}^{1/2}\, \mathrm{d}E_{\mathrm{cm}}, \tag{1}$$

where m_μ is the effective mass and E_{cm} is the mass of the centre of mass of the electron–hole system. The energy argument of the delta function must also include the phonon energy $\hbar\nu_{\mathrm{ph}}$

and the centre-of-mass energy such that

$$\varepsilon - E_{\mathrm{g}} - N\hbar\omega \rightarrow \varepsilon - E_{\mathrm{g_i}} - E_{\mathrm{cm}} - N\hbar\omega \pm h\nu_{\mathrm{ph}},$$

where $\pm\hbar\nu_{\mathrm{ph}}$ stand for the emission or absorption of a phonon. It is also necessary to replace the direct transition matrix element with the indirect element, include the phonon Boltzmann factor and sum over all CM degrees of freedom [9]. We then get

$$\alpha_{\mathrm{ind}}^{(1)}(\omega, F) = \frac{(2m_{\mu}^{3})^{1/2}}{2\pi^{2}\hbar^{3}} \int_{0}^{\infty} E_{\mathrm{CM}}^{1/2}\, \mathrm{d}E_{\mathrm{CM}}\alpha_{\mathrm{dir}}^{(1)}(\omega, F), \tag{2}$$

where $\alpha_{\mathrm{dir}}^{(1)}(\omega, F)$ is the direct single-photon absorption coefficient and F is the electric field. It is assumed that the matrix elements in the direct term must be modified to include the phonon matrix elements and the phonon occupation number.

It is straightforward to extend the above theory to the case of the N-photon process. We have already shown [11] that the N-photon process in a direct transition can be viewed as a single-photon process with an energy gap rescaled to E_{g}/N. This property gives a universal character to the FK effect and allows us to calculate the N-photon absorption coefficient for an indirect semiconductor as

$$\beta_{\mathrm{ind}}^{(2)}(\omega, F) = \frac{(2m_{\mu}^{3})^{1/2}}{2\pi^{2}\hbar^{3}} \int_{0}^{\infty} E_{\mathrm{CM}}^{1/2}\, \mathrm{d}E_{\mathrm{CM}}\beta_{\mathrm{dir}}^{(2)}(\omega, F), \tag{3}$$

where $\beta_{\mathrm{dir}}^{(N)}(\omega, F)$ is the direct N-photon absorption coefficient. Since the two-photon absorption is a two-step process where the electron jumps from the valence band into an intermediate conduction band (*virtual*) and finally into the conduction band, there are three types of two-photon transitions that must be considered depending upon the parity of the initial and final state: these are the a–a, a–f and f–f transitions. We now evaluate these cases for $N = 2$.

3. General two-photon absorption coefficient

From the initial work on double-photon transition by Braunstein [14, 22] and in band calculations that include exciton effects and band non-parabolicity [23–25], the two-photon absorption coefficient can be expressed in terms of a general equation that contains few fundamental constants that are almost the same for any semiconductor. In general, the expression can be simplified considerably if the values for the momentum matrix elements are extracted from band-structure model calculations based on Hartree–Fock calculations [26], where $p_{\mathrm{vc}}^{2}/m \approx 3E_{\mathrm{g}}/4m_{\mu}$. We then find that the two-photon absorption coefficient can be expressed as

$$\beta_{n}^{(2)}(\omega) = f_{n}\frac{K E_{\mathrm{p}}^{1/2}}{n_{\mathrm{o}}^{2}E_{\mathrm{g}}^{3}} F_{2}^{(n)}\left(\frac{\hbar\omega}{E_{\mathrm{g}}}\right), \tag{4}$$

where

$$F_{2}^{(n)}(x) = \frac{(2x-1)^{\frac{1}{2}+n}}{(2x)^{5}} \tag{5}$$

and $n = 0$, 1 or 2 for the a–a, a–f and f–f transitions, respectively; $f_{n} \leqslant 1$ is a numerical factor that depends on the angular averages of the interband matrix elements [26], which for practical applications can be used as a reasonable fitting parameter; $K = \frac{2^{9}\pi}{5}\frac{e^{4}}{\sqrt{m_{\mathrm{o}}}c^{2}}$, where e is the charge of the electron and m_{o} is its bare mass. The value for $K = 1940$ in units such that β is in cm/GW and E_{g} is in eV and $E_{\mathrm{p}} \approx 21$ eV for most semiconductors [27].

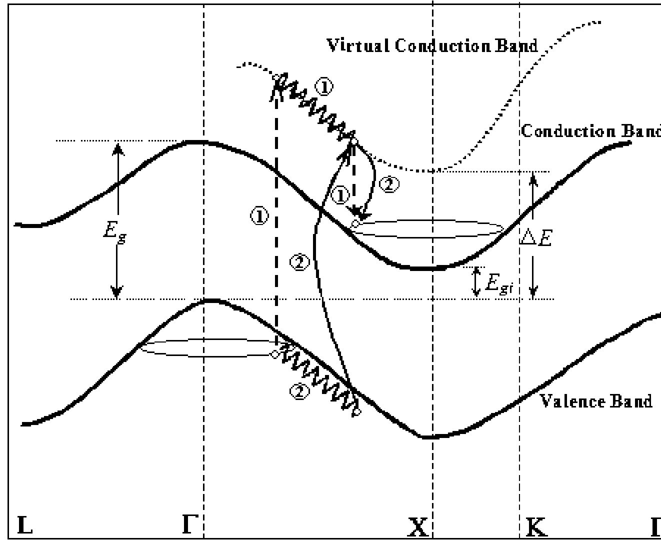

Figure 1. Schematic diagram of the band-structure model used to calculate the two-photon absorption coefficient for an indirect gap semiconductor. Two out of the many possible transitions are shown.

Out of all the possible transitions described above, the $f-f$ is the weakest one. Previous calculations using complicated perturbation theory [12] predicted this transition as a dominant contribution to two-photon absorption in general and in silicon in particular. However, we found from our calculations that the contribution of this $f-f$ transition to the two-photon process was negligible and likely reasons for the previous result of Dinu [12] were incorrect assumptions for the momentum matrix element and a mathematical error in the final results. The model that we have in mind is shown in figure 1. It consists of a valence band with a maximum located at $k = 0$, a virtual band located above the conduction band and a conduction band whose minimum is located at $k \neq 0$. This model we believe could be used to understand the 2-photon process in indirect band-gap semiconductors like Si. Using equation (4) in equation (3) and introducing generalized electron–phonon matrix elements given by [29]:

$$|H_{\mathrm{ep}}(Q)|^2 = |M_{\mathrm{ep}}|^2\left(\eta_Q + \tfrac{1}{2} \pm \tfrac{1}{2}\right), \tag{6}$$

where $|M_{\mathrm{ep}}|^2$ is the zero point electron–phonon square matrix elements which depends on the phonon branch (optical, acoustical), the $\pm$ stands for the absorption or emission of a phonon and η_Q is the phonon occupation number given by

$$\eta_Q = \frac{1}{\exp\left(\frac{E_{\mathrm{ph}}(Q)}{kT}\right) - 1}. \tag{7}$$

After Wedland [30], we know that the most important contribution to the indirect transition in Si is transverse-optical phonons, so our theory has this interaction in mind. We find that for any transition, the two-photon absorption coefficient for an indirect gap semiconductor in the absence of any dc-field can be expressed as

$$\beta^{(2)}_{\mathrm{ind},n}(\omega) = \frac{f_n\left(2m_\mu^3 E_{\mathrm{p}}\right)^{1/2}}{2\pi^2\hbar^3}\frac{K E_{\mathrm{gi}}^{\frac{3}{2}-n}}{n_{\mathrm{o}}^2(2\hbar\omega)^5}\left[|M_{\mathrm{ep}}|^2\left(\eta_Q + \frac{1}{2} \pm \frac{1}{2}\right)\right](2\hbar\omega - E_{\mathrm{gi}} \mp \hbar\nu_Q)^{2+n} \times \int_0^1 (1-x)^{\frac{1}{2}+n} x^{1/2}\,\mathrm{d}x, \tag{8}$$

where we have used E_{gi} to denote the indirect band-gap energy as shown in figure 1 and $n = 0$, 1 or 2 correspond to the a–a, a–f and f–f transitions, respectively.

3.1. Allowed–allowed (a–a) transition (F = 0)

In this case, the electron can jump into the intermediate conduction band vc_n with opposite parity as the valence band by first absorbing a photon, then emitting or absorbing a phonon and finally making the transition to the conduction band, which is of opposite parity to the intermediate conduction band. An alternate process is when the electron first emits or absorbs a phonon and then jumps into the intermediate band with opposite parity as the valence band and finally makes a transition to the conduction band. This process is depicted in figure 1. There is no intraband contribution because it is forbidden by parity conservation. The final two-photon absorption coefficient using equation (8) with $n = 0$ is

$$\beta^{(2)}_{\mathrm{ind,aa}}(\omega) = \frac{f_n m_\mu^{3/2}}{4\sqrt{2}\pi^2\hbar^3} \frac{E_{\mathrm{p}}^{1/2} K E_{\mathrm{gi}}^5}{E_{\mathrm{gi}}^{3/2} n_{\mathrm{o}}^2 (2\hbar\omega)^5} [|M_{\mathrm{ep}}|^2 (2\eta_Q + 1 \pm 1)] \times \left(\frac{2\hbar\omega}{E_{\mathrm{gi}}} - 1 \mp \frac{h\nu_Q}{E_{\mathrm{gi}}}\right)^2 \int_0^1 \sqrt{(1-x)\cdot x}\,\mathrm{d}x. \tag{9}$$

The integral in the above equation has a value of 0.393. If we express all the coefficients as a single constant called C_2 we get

$$\beta^{(2)}_{\mathrm{ind,aa}}(\omega) = C_2 F^{(2)}_{\mathrm{aa}}\left(\frac{\hbar\omega}{E_{\mathrm{gi}}}\right), \tag{10}$$

where

$$F^{(2)}_{\mathrm{aa}}(y) = \frac{\left(2y - 1 \mp \frac{\hbar\nu_Q}{E_{\mathrm{gi}}}\right)^2}{(2y)^5} \int_0^1 (1-x)^{1/2} x^{1/2}\,\mathrm{d}x \tag{11}$$

and

$$C_2 = \frac{f_n m_\mu^{3/2}}{4\sqrt{2}\pi^2\hbar^3} \frac{E_{\mathrm{p}}^{1/2} K}{E_{\mathrm{gi}}^{3/2} n_{\mathrm{o}}^2} [|M_{\mathrm{ep}}|^2 (2\eta_Q + 1 \pm 1)]. \tag{12}$$

The function in equation (11) has a maximum value at $\hbar\omega \cong (5/6)(1 \pm (h\nu_Q/E_{\mathrm{gi}}))E_{\mathrm{gi}}$ in contrast to the case of the a–a transition for a direct gap semiconductor where the maximum is located at 5/9 of E_{g}.

3.2. Allowed–forbidden (a–f) transition (F = 0)

This transition can occur through different routes: the electron can jump into the virtual conduction band with opposite parity and decay into the conduction band with equal parity (*which is the forbidden step here*); alternately, it can jump into the conduction band and perform an intraband transition which is forbidden. The two-photon absorption coefficient for this process can be expressed in a similar form as the previous case given by

$$\beta^{(2)}_{\mathrm{ind,af}}(\omega) = C_2 F^{(2)}_{\mathrm{af}}\left(\frac{\hbar\omega}{E_{\mathrm{gi}}}\right), \tag{13}$$

where

$$F^{(2)}_{\mathrm{af}}(y) = \frac{\left(2y - 1 \mp \frac{\hbar\nu_Q}{E_{\mathrm{gi}}}\right)^3}{(2y)^5} \int_0^1 (1-x)^{3/2} x^{1/2}\,\mathrm{d}x \tag{14}$$

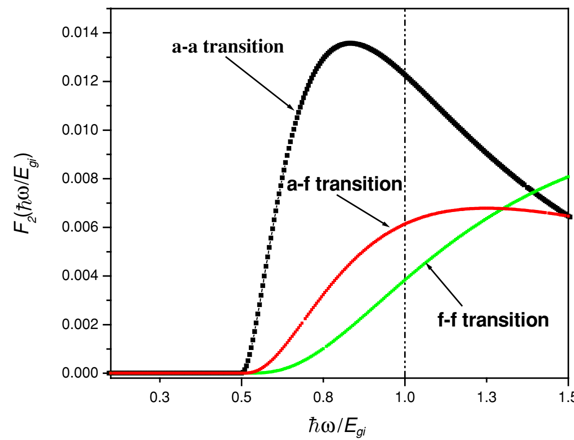

Figure 2. Dispersion curves for phonon-assisted two-photon absorption. The maximum of the two-photon absorption coefficient occurs above the indirect band-gap for a–f and f–f transitions, while it occurs below the indirect gap for a–a transitions.

and C_2 is given by equation (12). The integral in equation (14) has a value of 0.196. Equation (13) is of the same form as was determined in [13, 19]. The two-photon absorption coefficient has a maximum slightly above the indirect band-gap, at $\hbar\omega \approx (5/4)E_{\text{ig}}(1 \pm (h\nu_Q/E_{\text{gi}}))$.

3.3. *Forbidden–forbidden (f–f) transition (F = 0)*

There are several possible processes for this transition: one is where the electron can undergo intraband and interband transitions where all the bands have the same parity. The results from equation (8) can be expressed as

$$\beta^{(2)}_{\text{ind,ff}}(\omega) = C_2 F^{(2)}_{\text{ff}}\left(\frac{\hbar\omega}{E_{\text{gi}}}\right), \tag{15}$$

where

$$F^{(2)}_{\text{ff}}(y) = \frac{\left(2y - 1 \mp \frac{\hbar\nu_Q}{E_{\text{gi}}}\right)^4}{(2y)^5} \int_0^1 (1-x)^{5/2} x^{1/2}\, \mathrm{d}x. \tag{16}$$

This equation has the similar spectral dependence as that found in [12]. However, there are a few comments in this regard. First in [12] there was a mistake in the power dependence of the indirect gap. Second, as shown in figure 2 the maximum occurs at $\hbar\omega \approx (5/2)E_{\text{ig}}(1 \pm (h\nu_Q/E_{\text{gi}}))$ and not at $7E_{\text{gi}}/6$. This is because the denominator has a fifth power dependence and not a seventh. Finally, the integral in equation (16) has a value of 0.123 which makes the contribution smaller compared to the a–a and a–f transitions. This is shown in figure 2 where we have plotted the function $F^{(2)}_{\text{ii}}(y)$ versus the normalized energy $\hbar\omega/E_{\text{gi}}$. As can be clearly seen, the f–f is the weakest and only starts to dominate at a point where the single-photon process is dominant.

The above set of calculations, which are simple in principle, give us a unified picture of two-photon absorption for indirect gap semiconductors. Introducing more complex calculations could refine the expressions, but the final results will have similar spectral dependences. Below, we extend the theory to include the effect of a strong dc-electric field parallel to the direction of propagation of the optical field. We refer to this as nonlinear electro-absorption or phonon-assisted two-photon electro-absorption.

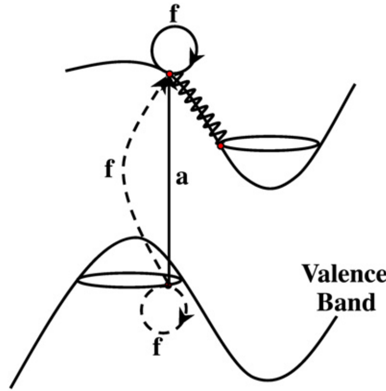

Figure 3. Schematics of the simplest two-step transitions leading to the net interband excitation.

4. Electro-absorption for an indirect gap semiconductor

With the formalism developed in the previous sections we can calculate the absorption coefficient in the presence of a strong electric field. Equation (3) requires knowing the electro-absorption coefficient for a direct gap semiconductor. This coefficient can be obtained using the standard dressed state calculation if the theory is modified to account for the different types of transitions that the electron goes through before it ends in the conduction band. The dressed state calculation can account for a–f and f–f transitions if suitable momentum matrix elements are used in the theory. The reason why the dressed state calculation can account for a–f and f–f is because it takes into account only two bands (valence and conduction) and as is well known, an intraband transition is always forbidden. Therefore, the only possible combination for the two-step process is to go from a valence band to the conduction band in an allowed or forbidden transition and then undergo an intraband transition that is forbidden; figure 3 shows a schematic of the transition schemes that dressed state calculations can account for. Using dressed state calculations Bahae e*t al* [27] showed that the two-photon absorption coefficient has a functional form given by

$$\beta^{(2)}(\omega) = K\frac{\sqrt{E_{\rm p}}}{n_{\rm o}^2 E_{\rm g}^3} F_2\left(\frac{\hbar\omega}{E_{\rm g}}\right), \tag{17}$$

where

$$F_2(x) = \frac{(2x-1)^{3/2}}{(2x)^5}.$$

If equation (17) is used in conjunction with equation (3) and assuming that: (i) you have an interband allowed process; (ii) an intraband or self-transition that is forbidden and (iii) the momentum matrix elements are independent of the electron momentum and can be approximated as

$$\frac{|P_{\rm vc}|^2}{m_{\rm o}^2} = \frac{3E_{\rm g}}{m_\mu}$$

then, our results are identical to that of Yee and Chau [13]. On the other hand, if the interband transition is forbidden, one may assume that the momentum matrix elements are proportional to electron momentum $\boldsymbol{k_i}$ of the initial state given by

$$|M_{\rm vc}|^2 = \left(\frac{m_{\rm o}}{m_\mu}\right)^2 \hbar^2 (e_q\cdot k_i)^2,$$

where $\boldsymbol{e_q}$ represents a unit vector for the photon polarization. In this case, we obtain results similar to Dinu [12] for the case of the f–f transition. Moreover, if in the dressed state calculation used by Keldysh [6] or Bahae [27] we introduce phenomenological dressed matrix elements given by

$$P_{\rm vc}^{\rm Dress} = P_{\rm vc}\frac{|M_{\rm vc}|^n}{M_{\rm vc}}, \tag{18}$$

where n is 0, 1, 2 for a–a, a–f, f–f, respectively, then the dressed state calculation can account for all the possible transitions in a three-band model and can be easily extended to the case where the electric field is present. This modification not only accounts for the direct case, but also using equation (3) for the indirect case. We can view equation (18) as compensating for the deficiency in the two-band model by introducing a strength factor that accounts for other possible transitions. With this approximation the two-photon absorption coefficient for an indirect semiconductor in the presence of a dc-electric field is given, in general, by [10, 11]

$$\beta_{{\rm ind},n}^{(2)}(\omega, F) = f_n^{\rm Dress} C_2 E_\mu^{2+n} \int_{\varepsilon_{\rm o}}^{\infty} t^{1/2}\,{\rm d}t \int_{\varepsilon_{\rm o}+t}^{\infty} (\varepsilon - t - \varepsilon_{\rm o})^n |{\rm Ai}(\varepsilon)|^2\,{\rm d}\varepsilon, \tag{19}$$

where $E_\mu = \left(\frac{\hbar^2 e^2 F^2}{2m_\mu}\right)^{1/3}$ is the characteristic energy of the electric field, $t = E_{\rm CM}/E_\mu$, C_2 is given by equation (12), $f_n^{\rm Dress}$ is a constant with values of π, $3\pi/2$ and $15\pi/8$ for a–a, a–f and f–f, respectively, ${\rm Ai}(x)$ is the Airy function and $\varepsilon_{\rm o}$ are given by

$$\varepsilon_{\rm o} = \frac{E_{\rm gi} \mp \hbar\nu_Q - 2\hbar\omega}{E_\mu}$$

Equation (19) can be readily evaluated using any computer-based mathematical routine. While the electric field introduces more complexity in the energy dependence of the nonlinear absorption coefficient, the expressions for any of the specific transitions (a–a, a–f or f–f) with or without the E-field will only differ from each other by some constant factor.

As mentioned in the introduction, the effect of an electric field in solids is the appearance of an absorption tail below the fundamental absorption edge (tunnelling) and oscillatory behaviour above it [6, 7]. We have shown that this effect can have universal character if the band edge is rescaled to an energy given by $E_{\rm g}/N$ where N is the number of photons absorbed in the process. However, it is remarkable that in the case of indirect absorption some of the features of the Franz–Keldysh effect disappear. To see this, we have plotted in figure 4, the rescaled electro-optic function defined below versus the photon energy for silicon in a field of $F = 1.8 \times 10^6$ V cm^{-1}. The rescaled electro-optic function is the difference of the two-photon absorption coefficient in the presence and absence of the electric field and is given by

$$\Delta\beta_{{\rm ind},n}^{(2)}(\omega, F) = \beta_{{\rm ind},n}^{(2)}(\omega, F) - \beta_{{\rm ind},n}^{(2)}(\omega, 0)$$

As can be seen from figure 4, the only process that shows oscillatory behaviour is the a–a transition despite the fact that all of them show exponential decay of the nonlinear absorption below the rescaled energy gap.

Figure 5 shows the dependence of the electro-optic function for different values of the electric field for the case of a–a transition. The strong tunnelling behaviour of the transitions is of particular interest for device design; it is seen that electro-optics devices based on silicon will have large two-photon absorption even below half of the indirect energy gap.

5. Proposed experiment and summary

To test experimentally some of the conclusions of the above model, we are proposing a modification of recent experimental development based on white-light continuum Z-scan [28]. In our case, the applied dc-field to the sample will be modulated with a low frequency that can

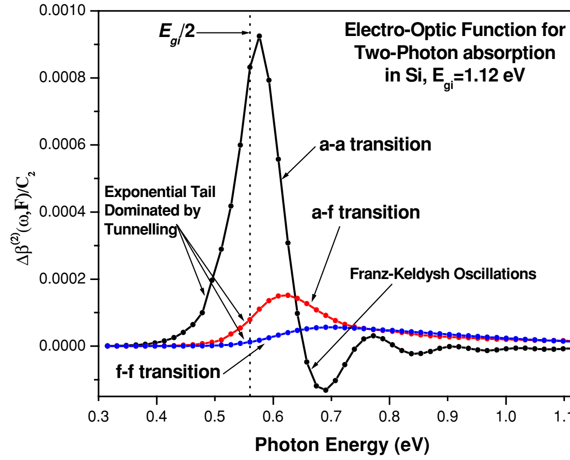

Figure 4. Two-photon electro-optic function defines as $\Delta\beta^{(2)}(\omega, F) = [\beta^{(2)}(\omega, F)\beta^{(2)}(\omega, F = 0)]/\beta^{(2)}(\omega, F = 0)]$, for Si, as a function of photon energy for a $F = 1.8 \times 10^6$ V cm^{-1} dc-field. The non-oscillatory behaviour of the a–f and f–f transitions above the energy gap are clearly evident.

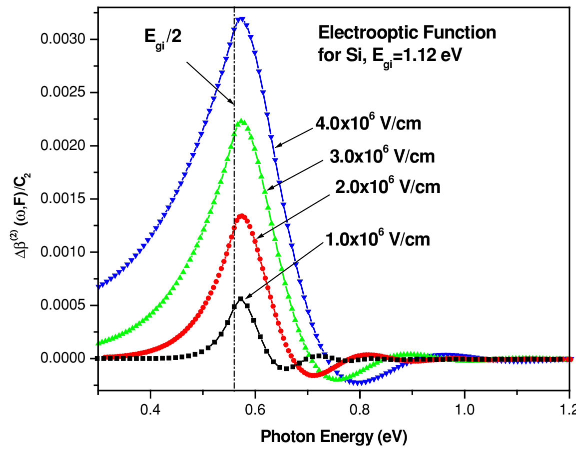

Figure 5. Scaled electro-optic function for Si under the influence of several dc-field strengths.

be used as an input reference to a lock-in amplifier. This will increase signal sensitivity and will allow detection of small variations in the absorption coefficient as a function of the dc-electric field. The experiment can be done in a single wavelength setup under variable electric field strength, or it can be done in a multi-wavelength setting using white-light continuum with wavelength selection achieved with a commercially available linear variable frequency (LVF) bandpass thin-film filter as was suggested in [28].

In conclusion, we have calculated the two-photon absorption coefficient for an indirect gap semiconductor. The result shows agreement with previous calculation for the case of a–f and f–f transitions. We extended the formalism to the case of a–a transition and generalized the results when a dc-field is present perpendicular to the layer. One of the results from our analysis is that the transitions have behaviour similar to the Franz–Keldysh effect

below the rescaled energy gap, with strong tunnelling for all the transitions, but an intriguing disappearance of the FK oscillations above $E_{ig}/2$ edge for the $a-f$ and $f-f$ transitions. This absence of the oscillations in the allowed–forbidden and forbidden–forbidden transitions is possible due to the weak coupling of the tails of the electron and hole wavefunctions.

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